4 Mathematics

4.1 Learning objectives

After studying this text the learner should / should be able to:

- Define and explain the time value of money concept and its mathematical manifestation: present value / future value (PV / FV).
- Examine the mathematics of the money market, the basis of which is the PV / FV concept.
- Evaluate the two basic money market calculations: discount and coupon, and the other calculations that derive from these.

4.2 Introduction

The mathematics of the money market is straightforward. It is apparent that money has a value over time, and that the value of money changes all the time. It depends on various factors such as the rate of inflation and the demand for money and credit. Central to the value of money is what the central bank thinks it (i.e. the interest rate) should be. The central bank has substantive control over the time value of money via its cash reserves accommodation procedures and the interest rate it charges for this assistance (known as bank rate, discount rate, base rate, repo rate, etc.).

The time value of money is expressed as a rate of interest, and interest may be payable daily, monthly, quarterly, annually, at the end of a short period or at the end of a period of years. Interest may also be paid up-front (some believe). In the case of the money market, interest is usually payable in a period of less than a year.

The following is covered in this text:

- Time value of money concept.
- Simple interest.
- Compound interest.
- Broken periods of less than a year (one interest payment).
- Discount.
- Effective rate.
- Interest-add-on securities (also called coupon securities).
- Discount securities.
- Treasury bill tender mathematics.
- Bonds with less than six months to maturity date.
- Bonds with more than six months to maturity date.

4.3 Time value of money concept

The *time value of money* concept, which clearly means that *money has a value over time*, is founded on the basic fact that money represents a command over goods and services (i.e. consumption). If you delay consumption by lending part of your money supply to someone you will expect compensation; otherwise you would not lend the money. What's the point?

Even if you were inclined to lend the money to a friend compensation-free, this is a foolish idea, because the future is uncertain. There are two factors to consider in relation to the future: you cannot be certain that you will receive the money lent and/or the compensation amount when they are due (= credit risk), and inflation may erode the value of the money lent. The compensation amount is called interest.

Another way of looking at this concept is that LCC received today is worth more than LCC received at some stage in the future. Ask someone if they would like you to give them LCC 100 today or in three months' time? The answer is obvious. This is because the LCC can be invested and its value enhanced by the rate of return, the interest amount, and because of the reduced purchasing power of the money as a result of inflation.

This is the basic tenet of the time value of money concept, i.e. money has a *future value* and a *present value*: Future value is present value plus interest, and present value is future value discounted by an appropriate rate.

Another basic principle of the concept is that interest is compounded, i.e. interest that is earned is reinvested, and an essential assumption here is that interest earned is reinvested at the rate earned on the principal amount.

The present / future value concept is the foundation of virtually all financial market mathematics.

4.4 Simple interest

Every money transaction involves a lender and a borrower. The lender (or surplus economic unit) lends, while the borrower (or deficit economic unit) borrows, at a rate of interest in order to compensate the lender for the risk s/he is taking. Clearly, the word *investment* applies to the lender while the word *borrowing* applies to the borrower. Simple interest is merely the interest payable at the end of the term of investment or borrowing. The formula used in the calculation of simple interest is as follows:

$$IA = PA \times i \times t$$

where

IA = interest amount
PA = principal amount (i.e. the amount invested / borrowed)
i = interest rate per time period expressed as a part of 1 (e.g. 0.12)
t = term or time (period/s for which interest is to be earned / paid).

An example follows:

PA = LCC 1 000 000
i = 12%
t = one year.

In this example the interest payable on the investment / borrowing is LCC 120 000 and it is calculated as follows:

IA = LCC 1 000 000 × 0.12 × 1
= LCC 120 000.

If the term is two years, and the 12% interest rate is payable at the end of the period (i.e. for two years), the interest amount would be the same as above, i.e. LCC 120 000. However, if the interest rate is payable *per annum*, i.e. for two time periods of a year each, the interest amount is LCC 240 000.

It will also be apparent that this formula may be expressed in terms of the present value (PV) and the future value (FV) of money. The principal value (amount invested) is the PV and the FV is the sum of the PV and the interest amount, as follows:

$$FV = PV + IA$$

or

$$FV = PV + (PV \times i \times t)$$

or

$$FV = PV \times [1 + (i \times t)].$$

This formula is usually written as:

$$FV = PV\ [1 + (i \times t)].$$

An example follows:

PV = LCC 1 000 000
i = 15% per annum (pa)
t = two years

FV = LCC 1 000 000 [1 + (0.15 × 2)]
= LCC 1 000 000 (1.3)
= LCC 1 300 000.

The PV may be derived from the FV formula as follows:

$$PV = FV / [1 + (i \times t)].$$

An example follows:

FV = LCC 1 500 000
i = 13% pa
t = one year

FV = LCC 1 500 000 / [1 + (0.13 × 1)]
= LCC 1 500 000 / 1.13
= LCC 1 327 433.63.

4.5 Compound interest

Compound interest takes into account interest earned on interest. It assumes always that the interest earned is reinvested at the original rate of interest from as soon as it is paid. The compound interest formula may be presented as follows:

$$FV = PV\ (1 + i/cp)^{y.cp}$$

where

i = interest rate pa, expressed as a unit of 1
y = number of years
cp = coupon periods (number of times interest is paid pa).

An example follows:

PV = LCC 1 000 000
i = 15% pa
y = 1
cp = 12 (i.e. monthly)

$$FV = \text{LCC } 1\ 000\ 000\ (1 + 0.15/12)^{1x12}$$
$$= \text{LCC } 1\ 000\ 000\ (1.0125)^{12}$$
$$= \text{LCC } 1\ 000\ 000\ (1.16075452)$$
$$= \text{LCC } 1\ 160\ 754.52.$$

Another example:

PV = LCC 1 000 000
i = 15%
y = 3
cp = 2 (i.e. six-monthly)

$$FV = \text{LCC } 1\ 000\ 000\ (1 + 0.15/2)^{3\times 2}$$
$$= \text{LCC } 1\ 000\ 000\ (1.075)^{6}$$
$$= \text{LCC } 1\ 000\ 000\ (1.54330153)$$
$$= \text{LCC } 1\ 543\ 301.53.$$

Yet another example:

PV	= LCC 1 000 000
i	= 12% pa
y	= 2
cp	= 1 (i.e. one payment per year, in arrears)

$$
\begin{aligned}
FV &= \text{LCC } 1\,000\,000\,(1 + 0.12/1)^{2\times 1} \\
&= \text{LCC } 1\,000\,000\,(1.12)^{2} \\
&= \text{LCC } 1\,000\,000\,(1.25440) \\
&= \text{LCC } 1\,254\,400.00.
\end{aligned}
$$

The principal of compound interest should be apparent from the last example. Interest of LCC 120 000 (0.12 × LCC 1 000 000) is earned at the end of the first year and at the end of the second year. But the first interest payment of LCC 120 000 is invested (at the same rate it is assumed) for the balance of the period of the investment (i.e. 1 year). This amount earns interest of LCC 14 400.00 (0.12 × LCC 120 000). Thus, the total amount of interest earned is LCC 254 400.00 (LCC 120 000 + LCC 120 000 + LCC 14 400.00). The sum of the total amount of interest earned and the original investment (PV) is the FV.

4.6 Broken periods of less than a year (one interest payment)

As seen, rates of interest are usually expressed in per annum terms. If broken periods of a year are introduced and one interest payment is made at the end of the period, the formulae become (t = number of days to maturity / 365):

FV = PV [1 + (i × t)]

PV = FV / [1 + (i × t)].

An example:

PV = LCC 1 000 000
i = 14% pa
t = 90 / 365

FV = = PV [1 + (i × t)]
= LCC 1 000 000 [1 + (0.14 × 90/365)]
= LCC 1 000 000 (1.03452)
= LCC 1 034 520.55.

Another example:

FV = LCC 1 350 000
i = 12% pa
t = 120 / 365

PV = FV / [1 + (i x t)]
= LCC 1 350 000 / [1 + (0.12 × 120/365)]
= LCC 1 350 000 / (1.03945)
= LCC 1 298 761.20.

A name for the above is "interest add-on", and if the above was a deposit, then its maturity value (= future value) was calculated. This type of security (e.g. NNCD or NCD) is called an "interest add-on security".

4.7 Discount

In the above examples it was assumed that interest is payable at the end of the period. In many cases in the money market interest is payable "up-front" (it is not really so as we shall see), meaning that the securities are issued and traded on a *discount basis*.

Thus, as opposed to interest-add-on securities where the amount invested is given (PV) and the interest factor is added to determine the future value FV), in the case of discounted securities the *given amount* is the *future value* (also called the face value or nominal value) and this amount is discounted to determine the present value (the amount to be paid, or its value *now* in the case of an existing security).

With discounted securities the interest amount paid "up-front" is called the discount and the amount paid by the investor or purchaser is called the discounted value. As noted above, the face value is the future value (FV) and the amount paid (the discounted value) the PV. The difference is the discount. The formulae are as follows:

$$D = FV \times d \times t$$

where

D = discount amount
FV = face value (or future value or nominal value)
d = discount rate pa (expressed as a part of 1)
t = term to maturity in days / 365.

The discounted value (or present value) is:

$$PV = FV - D$$

or

$$PV = FV - (FV \times d \times t)$$

or

$$PV = FV\,[1 - (d \times t)].$$

An example follows:

FV = LCC 1 000 000
d = 11.0% pa
t = 91 / 365

$$
\begin{aligned}
PV \quad &= PV \quad = FV\,[1 - (d \times t)] \\
&= LCC\ 1\ 000\ 000\ [1 - (0.11 \times 91/365)] \\
&= LCC\ 1\ 000\ 000\ (1 - 0.02742466) \\
&= LCC\ 1\ 000\ 000\ (0.97257534) \\
&= LCC\ 972\ 575.34.
\end{aligned}
$$

From the above it will be evident that there is a fundamental difference between discount and interest. An interest amount is based on the PV, and the FV is the sum of the two. The discount amount, on the other hand, is based on the FV, and the PV is the difference between the two. It follows that the rate of interest is always expressed as a percentage of the PV, while the discount rate is expressed as a percentage of the FV. The conversion formulae are as follows:

Interest to discount:

$$d \quad = i / [1 + (i \times t)].$$

Discount to interest:

$$i \quad = d / [1 - (d \times t)].$$

These formulae are important because in most money markets discount securities are traded on a yield basis and then converted to a discount in order to determine the consideration (PV) payable. An example follows:

$$d = 11.0\% \text{ pa}$$
$$t = 91 / 365$$

$$i = d / [1 - (d \times t)]$$
$$= 0.11 / [1 - (0.11 \times 91/365)]$$
$$= 0.11 / 0.972575$$
$$= 11.31\% \text{ pa.}$$

It should be apparent that when one has the yield (as here, i = 11.310182%), the usual formulae apply (use data of last example and enough decimals and compare with it):

$$PV = FV / [1 + (i \times t)]$$
$$= \text{LCC } 1\ 000\ 000 / 1.028198$$
$$= \text{LCC } 972\ 575.34.$$

$$FV = PV [1 + (i \times t)]$$
$$= \text{LCC } 972\ 575.34 \times 1.028198$$
$$= \text{LCC } 1\ 000\ 000.00.$$

4.8 Effective rate

Rates of interest pa in the financial markets are quoted with the interest frequency stated. These rates are referred to as the *nominal* rates. For example, a rate may be quoted as 13.5% pa with interest payable *monthly*, or a rate may be quoted as 12% pa with interest payable *quarterly*.

The terminology used in the market for these two rates are 13.5% *nacm* (nominal annual compounded monthly) and 12% *nacq* (nominal annual compounded quarterly). In the case where interest is payable six-monthly and at the end of a year, the terminology would be *nacs* (nominal annual compounded semi-annually) and *naca* (nominal annual compounded annually).

In order to compare these rates, the term *effective rate* is applied. *Nominal rates* are converted to *effective rates* with the use of the following formula:

$$i_e = [(1 + i_n / t)^t - 1]$$

where

i_e = effective rate
i_n = nominal rate
t = number of interest periods per annum.

An example: a 12% *nacm rate* converts to an *effective rate* as follows:

$$
\begin{aligned}
i_e &= (1 + i_n / t)^t - 1\\
&= (1 + 0.12/12)^{12} - 1\\
&= (1 + 0.01)^{12} - 1\\
&= 1.12683 - 1\\
&= 0.12683\\
&= 12.68\%.
\end{aligned}
$$

Another example: a 12% *nacq rate* converts to an *effective rate* as follows:

$$
\begin{aligned}
i_e &= (1 + i_n / t)^t - 1\\
&= (1 + 0.12/4)^{4} - 1\\
&= (1 + 0.03)^{4} - 1\\
&= 1.12550 - 1\\
&= 0.12550\\
&= 12.55\%.
\end{aligned}
$$

It will be evident that a 12% *naca* rate will be equal to an *effective rate* of 12%. Thus, the more interest period involved, the higher the effective rate will be.

The above formula may also be used for period of longer than a year where interest is payable at the end (it is covered later).

4.9 Interest-add-on securities

4.9.1 Introduction

The negotiable certificate of deposit (NCD) is the best example of an interest-add-on security. It can also be referred to as a coupon instrument, because a coupon is paid once or more frequently. There are a number of ways in which banks can issue negotiable certificates of deposit (NCDs). The main differentiation is short-term and long-term. Short-term refers to NCDs that are issued for periods of one year or shorter, and long-term to NCDs issued for periods longer than a year.

In the case of short-term NCDs interest is always payable at maturity (i.e. one coupon). In the case of long NCDs, interest may be payable at maturity (one coupon) or six-monthly in arrears (more than one coupon).

The following types of NCDs can be identified:

- Short NCDs where the amount invested is given
- Short NCDs where the maturity value is given
- Short NCDs issued at a discount to face value (in order to conceal the actual issue rate)
- Long NCDs with interest payable at maturity
- Long NCDs with interest payable six-monthly in arrears where the amount invested is given
- Long NCDs with interest payable six-monthly in arrears, issued at a discount to par value
- NCDs with a variable rate of interest.

The first-mentioned NCD is the most "common" type. Each method of issue and the mathematics pertaining to it is discussed below.

4.9.2 Short NCDs where the amount invested is given

This is the "common" NCD. A CD, which is simply a fixed deposit that is negotiable, is the most issued and traded money market instrument in the money market. At issue the typical simple interest calculation is involved, as follows:

FV = PV [1 + (i × t)].

An example will make this clear:

Amount of deposit (PV)	= LCC 1 000 000
Interest rate negotiated (i)	= 9.8% pa
Term required by depositor	= 180 days; therefore t = 180 / 365)

The *maturity value* is calculated by the deposit-taking bank and placed on the certificate. The maturity value is the FV, and the amount of the deposit the PV.

Maturity value (FV) = PV [1 + (0.098 x 180/365)]
= LCC 1 000 000 (1.04832877)
= LCC 1 048 328.77.

When NCDs are *traded* in the money market, the "givens" are:

- The maturity value (FV)
- The maturity date
- The settlement date
- The rate at which the trade takes place.

These variables are used to calculate the *consideration*, i.e. the amount to be paid by the purchaser (or received by the seller). The consideration is nothing else but the PV. The formula used in secondary market trades is as follows:

PV = FV / [1 + (i x t)].

An example: a company would like to invest an amount close to LCC 1 million and approaches its broker in this regard. The broker makes a few phone calls and offers the investor a NCD with the following characteristics:

Maturity value (FV)	= LCC 1 054 246.58 (this was calculated at issue)
Maturity (due) date	= 20 June 2002
Date of transaction	= 21 January 2002
Number of days	= 150 (i.e. 21 January to 20 June); therefore t = 150 / 365
Rate traded at (i)	= 9.2% pa.

The investor accepts the deal and the consideration is calculated:

Consideration (PV) = LCC 1 054 246.58 / [1 + (0.092 × 150/365)]
= LCC 1 054 246.58 / 1.03780822
= LCC 1 015 839.50.

4.9.3 Short NCDs where the maturity value is given

Occasionally, NCDs are issued where the maturity value is a given, i.e. the depositor wants to receive a certain amount at the end of the investment period (for example LCC 1 000 000). In this case the proceeds for the bank, i.e. the deposit amount, need to be calculated. The formula used is the PV formula as follows:

PV = FV / [1 + (i × t)].

An example is required. The depositor insists he would like to receive LCC 1 million at the end of the investment period that will suit him, which is 121 days. The bank negotiates a rate of 11.5% pa, which the depositor accepts. The "givens" are as follows:

Maturity value (FV) = LCC 1 000 000
Issue date = 1 April 2002
Maturity date = 31 July 2002
t = 121 / 365
i = 11.50% pa.

The amount of the deposit (i.e. the consideration, or PV) is calculated as follows:

Consideration (PV) = FV / [1 + (i × t)]
= LCC 1 000 000 / [1 + (0.115 × 121/365)]
= LCC 1 000 000 / 1.03812329
= LCC 963 276.72.

This calculation could also have been executed also by simply converting the issue rate (i) to a *discount rate* (d) as follows:

d = i / [1 + (i × t)]
= 0.115 / [1 + (0.115 × 121/365)]
= 0.115 / 1.03812329
= 0.11077682
= 11.077682% pa

The calculation is then the discount one:

$$\begin{aligned} PV &= FV\,[1 - (d \times t)] \\ &= LCC\ 1\ 000\ 000\ [1 - (0.11077682 \times 121/365)] \\ &= LCC\ 1\ 000\ 000 \times 0.96327672 \\ &= LCC\ 963\ 276.72. \end{aligned}$$

4.9.4 Short NCDs issued at a discount to face value (in order to conceal the issue rate)

At times NCDs are issued where the rate on which the maturity value is based and the actual issue rate are different. This is generally done when a bank wishes to conceal the actual issue rate from the ultimate investor. For example, if the bank is approached by a broker and bid for a NCD at a rate with which it is satisfied, it will not want the investor to know the issue rate because it will not know what "turn" is taken by the broker. The formula used to calculate the price is as follows:

$$\text{Price} = [1 + (r \times t)] / [1 + (i \times t)]$$

where

r = rate of interest that appears on the face of the certificate (the rate that determines the FV)
i = actual issue rate of interest % pa.

An example:

Issue date	= 1 April 2002
Maturity date	= 31 July 2002
t	= 121 / 365
r	= 11.50% pa
i	= 12.00% pa

Price = [1 + (0.115 × 121/365)] / [1 + (0.12 × 121/365)]
= (1.03812329) / (1.03978082)
= 0.99840589.

In this example the maturity value of a LCC 1 000 000 NCD (i.e. the stated amount of the deposit on the face of the NCD) will be LCC 1 038 123.29, but the proceeds to the bank, i.e. the actual amount of the deposit, will be LCC 998 405.89. The investor will not be able to determine the size of the "turn" taken by the broker.

The 12.0% pa rate at which the broker purchased the NCD may be verified by annualising in percentage pa terms the interest earned on the amount paid for the NCD:

Rate agreed = [(LCC 1 038 123.29 – LCC 998 405.89) / LCC 998 405.89] × 365/121
= (LCC 39 717.40 / LCC 998 405.89) × 365/121
= 12.0% pa.

4.9.5 Long NCDs with interest payable at maturity

As in the case of NCDs with tenors of one year or shorter, long NCDs can be issued with interest payable at maturity in the three forms as mentioned earlier.

In the case of a NCD issued for longer than one year with *interest payable at the end of the period,* the investor would want to know what his *effective rate* pa is. The formula used is the one employed above in the section on the *effective rate.* This is repeated here:

$$i_e = [(1 + i_n / t)^t - 1]$$

where

i_e = effective rate
i_n = nominal rate
t = number of interest periods per annum.

It should be noted here that t becomes a fraction of 1, i.e. the *number of interest periods per annum* is less that 1, and this is calculated by a year (365 days) over the term of the NCD. An example follows:

Issue date	= 1 April 2002
Maturity date	= 31 July 2003
Term to maturity	= 486
t	= 0.75102881 (365 / 486)
i_n	= 11.70%

$$
\begin{aligned}
i_e &= [(1 + i_n / t)^t - 1] \\
&= [(1 + 0.117 / 0.75102881)^{0.75102881} - 1] \\
&= (1.15578630)^{0.75102881} - 1 \\
&= 0.1149 \\
&= 11.49\%.
\end{aligned}
$$

4.9.6 Long NCDs with interest payable six-monthly in arrears where the amount invested is given

Interest on NCDs issued for longer than a year is usually payable six-monthly in arrears. An example is:

Amount of investment (PV)	= LCC 1 000 000
Issue date	= 15 October 2002
Redemption date	= 25 October 2004
Tenor	= 2 years and 10 days
Coupon rate	= 15.00% pa
Interest dates	= 25 October and 25 April.

In this example the first interest payment will be on 25 October 2002 and the amount will be LCC 4 109.59. This amount is calculated in terms of the simple interest formula shown earlier, as follows:

$$
\begin{aligned}
\text{Interest payment} &= PV\ (i \times t) \\
&= \text{LCC } 1\ 000\ 000\ (0.15 \times 10/365) \\
&= \text{LCC } 1\ 000\ 000\ (0.00410959) \\
&= \text{LCC } 4\ 109.59.
\end{aligned}
$$

Subsequent interest payments are, of course, equal to LCC 75 000, which is calculated in terms of the following formula:

$$
\begin{aligned}
\text{Interest payments} &= PV\ (i \times 1/2) \\
&= \text{LCC } 1\ 000\ 000\ (0.15 \times 0.5) \\
&= \text{LCC } 1\ 000\ 000\ (0.075) \\
&= \text{LCC } 75\ 000.
\end{aligned}
$$

4.9.7 Long NCDs with interest payable six-monthly in arrears, issued at a discount to par **value**

On occasions banks issue NCDs with interest payable six-monthly at a discount to nominal (also termed par or face) value (for example, LCC 1million). The formula used to calculate the price is the same as the one used to calculate the price of a long-term government or other bond, i.e. the price is equal to the present value of the future stream of interest payments plus the present value of the redemption proceeds (LCC 1 000 000) discounted at the actual rate at which the NCD is issued. An example will make this clear.

Nominal amount	= LCC 1 000 000
Issue date	= 30 April 2001
Redemption date	= 30 April 2004
Tenor	= 3 years
Coupon rate	= 13.0% pa
Actual issue rate	= 15.0% pa
Interest dates	= 30 April and 30 October.

In this example the price is calculated to be LCC 95.31%, and the amount received by the bank upon issue of the LCC 1 000 000 nominal value certificate is LCC 953 100.00.

4.9.8 NCDs with a variable rate of interest

At times banks issue NCDs for fixed periods, but the interest payable is variable. In this case the rate is specified with reference to some agreed benchmark rate. The rate may be determined daily or less frequently, depending on the requirement of the investor.

The valuation / pricing (after issue) mathematics in this regard is elaborate and will; not be discussed here.

4.10 Discount securities

Money market securities in most countries are negotiated (and traded) in the primary and secondary markets on an interest rate (yield) basis (in order to make comparisons easier), but the calculation of the consideration is effected on a discount basis where applicable: bankers' acceptances, treasury bills, commercial paper.

These instruments are issued with a nominal value (also termed par value, maturity value, end value and face value), which is actually the FV, equal to a round amount such as (and usually as) LCC 1 000 000 or multiples of this. When issued or sold, they are discounted at the discount rate that is derived from the interest rate negotiated, in order to arrive at the consideration (which is the PV).

The formulae that apply here were elucidated above and are repeated here for the sake of convenience.

$$PV = FV\,[1 - (d \times t)]$$

where

PV = present value (or the consideration)
FV = future value (also called face value / end value / nominal value)
d = discount rate pa (expressed as a part 1, e.g. 0.08)
t = number of days to maturity / 365.

An example: the central bank sells a LCC 100 million treasury bill to a bank for the purposes of monetary policy. The bill has 90 days to run (i.e. to maturity) and the discount rate (derived from the agreed yield) is 11.5% pa:

FV = LCC 100 000 000
d = 11.5% pa
t = 90 / 365

$$PV = FV\,[1 - (d \times t)]$$
$$= LCC\ 100\ 000\ 000\ [1 - (0.115 \times 90/365)]$$
$$= LCC\ 100\ 000\ 000\ (1 - 0.02835616)$$
$$= LCC\ 100\ 000\ 000\ (0.97164384)$$
$$= LCC\ 97\ 164\ 384.00.$$

What was the rate (i) that converted to a discount rate of 11.5%? As noted above, the conversion formula is as follows:

$$i = d / [1 - (d \times t)]$$
$$= 0.115 / [1 - (0.115 \times 90/365)]$$
$$= 0.115 / (1 - 0.02835616)$$
$$= 0.115 / 0.97164384$$
$$= 11.84\%\ pa.$$

As we saw earlier, converting from an interest rate to a discount rate involves the following formula:

$$d = i / [1 + (i \times t)].$$

4.11 Treasury bill tender mathematics

Central banks conduct the tender procedure for treasury bills on behalf of government. Most central banks issue 91-day and 182-day bills (and some for 273 days and 364 days) and most require tenders in prices of multiples of 0.005. If an investor wishes to acquire 91-day bills at the current market discount rate of, say 10.80% pa, s/he will calculate the price as follows:

$$Price = 1 - (0.108 \times 91/365)$$
$$= 1 - 0.02692603$$
$$= 0.97307397.$$

This may also be written as LCC 97.307397% (i.e. the price per LCC 100 nominal / face value). But the tenderer cannot submit such a price because it must be in multiples of 0.005. S/he will most likely decide upon a price of LCC 97.305%. This price converts to a discount rate as follows:

$$Discount\ rate = (100 - 97.305) \times 365/91$$
$$= 10.8096\%\ pa.$$

This converts to an interest (yield) rate of:

$$
\begin{aligned}
i &= d / [1 - (d \times t)] \\
&= 0.108096 / [1 - (0.108096 \times 91/365)] \\
&= 0.108096 / 0.973050 \\
&= 0.1111 \\
&= 11.11\% \text{ pa}
\end{aligned}
$$

4.12 Bonds with less than six months to maturity date

Bonds (with interest payable six-monthly) that have less than six months to maturity date are traded according to the formula that applies to the common NCD as illustrated above. However, in this case, the maturity value (FV) of the instrument (bond) is equal to the nominal / face value of the bond plus the final coupon payment. An example is required:

Nominal amount	= LCC 1 000 000
Issue date	= 30 April 2005
Redemption date	= 30 April 2008
Tenor	= 3 years
Coupon rate	= 13.0% pa
Interest dates	= 30 April and 30 October
Settlement date	= 1 March 2008
Days to maturity	= 60; therefore t = 60 / 365
Rate dealt at	= 9.25% pa.

The applicable formula will be recalled:

$$PV = FV / [1 + (i \times t)].$$

As noted, the FV will be the nominal / face value plus the final coupon payment, i.e. the amounts that will be received on the redemption date:

LCC 1 000 000 + LCC 65 000 (0.13/2 × LCC 1 000 000) = LCC 1 065 000.

The calculation follows:

$$
\begin{aligned}
\text{PV (consideration)} &= \text{LCC } 1\,065\,000 / [1 + (0.0925 \times 60/365)] \\
&= \text{LCC } 1\,065\,000 / 1.01520548 \\
&= \text{LCC } 1\,049\,048.71.
\end{aligned}
$$

4.13 Bonds with longer than six months to maturity date

Bonds (with interest payable six-monthly) that have longer than six months but less than one year to maturity date are regarded as money market instruments. Such bonds are traded using the bond formula and the bond "rate" (which is called the *yield to maturity* – ytm). This will be covered in a separate module.

4.14 Bibliography

Faure, AP, 2007. **The money market**. Cape Town: Quoin Institute (Pty) Limited.